**THE CHINESE UNIVERSITY OF HONG KONG**
**Department of Mathematics**
**MATH3280A Introductory Probability 2023-2024 Term 1**
**Homework Assignment 1**
**Due Date: 18 September, 2023 (Monday)**

## General Regulations

- All assignments will be submitted and graded on CUHK Blackboard. You can view your grades and submit regrade requests here as well.

- Late assignments will receive a grade of 0.

- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

- Write your solutions on A4 white paper. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Failure to comply with these instructions will result in a 10-point deduction).

- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

1. A die is tossed twice and the number of dots facing up in each toss is counted and noted in the order of occurrence.

   (a) Find the sample space.
   (b) Find the set $A$ corresponding to the event *number of dots in first toss is not less than number of dots in second toss.*
   (c) Find the set $B$ corresponding to the event *number of dots in first toss is 6.*
   (d) Does $A$ imply $B$ or does $B$ imply $A$?
   (e) Find $A \cap B^c$ and describe this event in words.
   (f) Let $C$ correspond to the event *number of dots in dice differs by 2.* Find $A \cap C$.

2. Suppose that $A$ and $B$ are mutually exclusive events for which $P(A) = .35$ and $P(B) = .54$. What is the probability that

   (a) either $A$ or $B$ occurs?
   (b) $A$ occurs but $B$ does not?
   (c) both $A$ and $B$ occur?

3. A total of 28 percent of Hong Kong males smoke cigarettes, 8 percent smoke cigars, and 6 percent smoke both cigars and cigarettes.

   (a) What percentage of males smokes neither cigars nor cigarettes?
   (b) What percentage smokes cigars but not cigarettes?

4. Five people, designated as Adin, Bob, Carl, Dan, and Eddy, are arranged in linear order. Assuming that each possible order is equally likely, what is the probability that

   (a) there is exactly one person between Adin and Bob?
   (b) there are exactly two people between Adin and Bob?
   (c) there are exactly three people between Adin and Bob?

5. At a certain university, every year 8 to 12 students are granted University Best Student Awards. This year among the nominated students are Mary, Peter, and David. Let $A$, $B$, and $C$ denote the events, respectively, that these students will be given awards. In terms of $A$, $B$, and $C$, find an expression for the event that the award goes to

   (a) only Mary;
   (b) at least one of the three;
   (c) none of the three;
   (d) exactly two of them;
   (e) exactly one of them;
   (f) Mary or Peter but not both.

6. Let $A$ and $B$ be two events. Prove that

$$P(A \cap B) \geq P(A) + P(B) - 1.$$

7. Let $A$, $B$, and $C$ be three events. Show that exactly two of these events will occur with probability

$$P(A \cap B) + P(A \cap C) + P(B \cap C) - 3P(A \cap B \cap C).$$